# Statistical Simulation Models for Rayleigh and Rician Fading

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*Abstract*—**New simulation models are proposed for Rayleigh and Rician fading channels. First, the statistical properties of Clarke's fading model with a finite number of sinusoids are analyzed. An improved Clarke's model is then proposed for the simulation of Rayleigh fading channels. Based on this improved Rayleigh fading model, a novel simulation model is proposed for Rician fading channels. The new Rician fading model employs a *zero-mean* stochastic sinusoid as the specular (line-of-sight) component, in contrast to all existing Rician fading simulators that utilize a *non-zero mean* deterministic specular component. The statistical properties of the proposed Rician fading model are analyzed in detail. It is shown that the probability density function of the Rician fading phase is not only independent of time but also uniformly distributed over $[-\pi, \pi)$. This property is different from that of existing Rician fading models. The statistical properties of the new simulators are confirmed by extensive simulation results, finding good agreement with theoretical analysis in all cases. An explicit formula for the level crossing rate is derived for general Rician fading when the specular component has *non-zero* Doppler frequency.**

## I. Introduction

Mobile radio channel simulators are commonly used in the laboratory because they make system tests and evaluations less expensive and more reproducible than field trials. Many different techniques have been proposed for the modeling and simulation of mobile radio channels [1]-[16]. Among them, the well known Jakes' model [3], which is a simplified simulation model of Clarke's model [1], has been widely used for frequency nonselective Rayleigh fading channels for about three decades. Recently, various modifications [5], [9]-[12] and improvements [14], [16] of Jakes' simulator have been reported in the literature for generating multiple uncorrelated fading waveforms needed for frequency selective fading channels and multiple-input multiple-output (MIMO) channels. Since Jakes' simulator needs only one fourth the number of low-frequency oscillators than needed in Clarke's model, it is commonly perceived that Jakes' simulator (and its modifications) is more computationally efficient than Clarke's model. However, it was recently pointed out by Pop and Beaulieu in [13] that "reduction in the number of simulator oscillators based on azimuthal symmetries is meritless", and they proposed a Clarke's model-based simulator in [13]. In the first part of this paper, we give a statistical analysis of Clarke's model with a finite number of sinusoids and show that the simulator proposed in [13] has deficiencies in some of its higher-order statistics. We then propose an improved Clarke's model for the simulation of Rayleigh fading channels.

All the existing Rician channel models in the literature assume that the specular (line-of-sight) component is either *constant* and *non-zero* [7], or *time-varying* and *deterministic* [2], [9]. These assumptions may not reflect the physical nature of the specular components, particularly when the specular component is random, changing from time to time and from mobile to mobile. Furthermore, according to [2], all these Rician fading models are nonstationary in the wide sense and the probability density function (PDF) of the fading phase is a function of time [2], [9]. In the second part of this paper, a novel statistical simulation model will be proposed for Rician fading channels. The specular component will employ a *zero-mean* stochastic sinusoid with a pre-chosen angle of arrival and a random initial phase. This assumption implies that different specular components in different channels may have different initial phases.

The remainder of this paper is organized as follows. In Section II, we present the statistical properties of Clarke's model with a finite number of sinusoids and show that the model reported in [13] has limitations in its higher-order statistics. An improved Clarke's model for Rayleigh fading channels is proposed. In Section III, we present a novel statistical simulation model for Rician fading channels, and analyze the statistical properties of the new Rician fading model. Section IV gives extensive performance evaluations of the new Rayleigh and Rician fading simulators. Section V concludes the paper.

## II. An Improved Rayleigh Fading Simulator

Clarke's Rayleigh fading model is sometimes referred to as a mathematical reference model, and is commonly considered as a computationally inefficient model compared to Jakes' Rayleigh fading simulator. In this section, we show that Clarke's model with a finite number of sinusoids can be directly used for Rayleigh fading simulation, and that its computational efficiency and second-order statistics are as good as those of improved Jakes' simulators. We then briefly show that the model described in [13] contains higher-order statistical deficiencies and better the model by introducing randomness to the angle of arrival, which leads to improved higher-order statistics.

### *A. Clarke's Rayleigh Fading Model*

The baseband signal of the normalized Clarke's 2-D isotropic scattering Rayleigh fading model is given by [1], [19]

$$g(t) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \exp[j(w_d t \cos \alpha_n + \phi_n)], \tag{1}$$

where $N$ is the number of propagation paths, $w_d$ is the maximum radian Doppler frequency and $\alpha_n$ and $\phi_n$ are, respectively, the angle of arrival and initial phase of the $n$th propagation path. Both $\alpha_n$ and $\phi_n$ are uniformly distributed over $[-\pi, \pi)$ for all $n$ and they are mutually independent.

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The central limit theorem justifies that the real part, $g_c(t) = \mathrm{Re}[g(t)]$, and the imaginary part, $g_s(t) = \mathrm{Im}[g(t)]$, of the fading $g(t)$ can be approximated as Gaussian random processes for large $N$. Some desired second-order statistics for fading simulators are manifested in the autocorrelation and cross-correlation functions which are given in [19] for the case when $N$ approaches infinity. However, the statistical properties of Clarke's model with a finite $N$ (number of sinusoids) are not available in the literature. These properties are very important for justifying the suitability of Clarke's model as a valid Rayleigh fading simulator. Thus, we present some of these key statistics here.

*Theorem 1:* The autocorrelation and cross-correlation functions of the quadrature components, and the autocorrelation functions of the complex envelope and the squared envelope of fading signal $g(t)$ are given by

$$
\begin{aligned}
R_{g_c g_c}(\tau) &= E[g_c(t)g_c(t+\tau)] = \frac{1}{2}J_0(w_d\tau) && \text{(2a)}\\
R_{g_s g_s}(\tau) &= \frac{1}{2}J_0(w_d\tau) && \text{(2b)}\\
R_{g_c g_s}(\tau) &= 0 && \text{(2c)}\\
R_{g_s g_c}(\tau) &= 0 && \text{(2d)}\\
R_{gg}(\tau) &= E[g^*(t)g(t+\tau)] = J_0(w_d\tau) && \text{(2e)}\\
R_{|g|^2|g|^2}(\tau) &= 1 + J_0^2(w_d\tau) + \frac{1}{N}, && \text{(2f)}
\end{aligned}
$$

where $E[\cdot]$ denotes expectation and $J_0(\cdot)$ is the zeroth-order Bessel function of the first kind [18].

*Proof:* Omitted for brevity. ■

In simulation practice, time-averaging is often used in place of ensemble averaging. For example, the autocorrelation of the real part of the fading signal for one trial is given by

$$
\begin{aligned}
\hat{R}_{g_c g_c}(\tau) &= \lim_{T\to\infty}\frac{1}{T}\int_0^T g_c(t)g_c(t+\tau)dt\\
&= \frac{1}{2N}\sum_{n=1}^{N}\cos(w_d\tau\cos\alpha_n).
\end{aligned}
$$

Clearly, this time averaged autocorrelation changes from trial to trial due to random angle of arrival. Note that the variance of the time average $\mathrm{Var}\{R_{g_c g_c}(\tau)\} = E\left[|\hat{R}_{g_c g_c}(\tau) - 0.5J_0(w_d\tau)|^2\right]$, carries important information indicating the closeness between a single trial with finite $N$ and the ideal case with $N = \infty$. We now present the time-averaged variances of the aforementioned correlation statistics.

*Theorem 2:* The variances of the autocorrelation and cross-correlation of the quadrature components, and the variance of the autocorrelation of the complex envelope of the fading signal $g(t)$ are given by

$$
\begin{aligned}
\mathrm{Var}\{R_{g_c g_c}(\tau)\} &= \frac{1 + J_0(2w_d\tau) - 2J_0^2(w_d\tau)}{8N} && \text{(3a)}\\
\mathrm{Var}\{R_{g_s g_s}(\tau)\} &= \frac{1 + J_0(2w_d\tau) - 2J_0^2(w_d\tau)}{8N} && \text{(3b)}\\
\mathrm{Var}\{R_{g_c g_s}(\tau)\} &= \frac{1 - J_0(2w_d\tau)}{8N} && \text{(3c)}\\
\mathrm{Var}\{R_{g_s g_c}(\tau)\} &= \frac{1 - J_0(2w_d\tau)}{8N} && \text{(3d)}\\
\mathrm{Var}\{R_{gg}(\tau)\} &= \frac{1 - J_0^2(w_d\tau)}{N}. && \text{(3e)}
\end{aligned}
$$

*Proof:* Omitted for brevity. ■

As can be seen from Theorems 1 and 2, Clarke's model using a number of sinusoids, $N \geq 8$, can be usefully employed as a Rayleigh fading simulator. Its computational efficiency and statistics are similar to those of the recently improved Jakes models [14], [16], which have removed some statistical deficiencies of Jakes' original model [3] and various modified Jakes' models proposed in [5], [9]-[12].

### *B. Pop and Beaulieu's Simulator*

Based on Clarke's model given by (1), Pop and Beaulieu [12], [13] recently developed a Rayleigh fading simulator by setting $\alpha_n = \frac{2\pi n}{N}$ in $g(t)$. Thus, the lowpass fading process becomes

$$
\begin{aligned}
X(t) &= X_c(t) + jX_s(t) && \text{(4a)}\\
X_c(t) &= \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\cos\left(w_d t\cos\frac{2\pi n}{N} + \phi_n\right) && \text{(4b)}\\
X_s(t) &= \frac{1}{\sqrt{N}}\sum_{n=1}^{N}\sin\left(w_d t\cos\frac{2\pi n}{N} + \phi_n\right). && \text{(4c)}
\end{aligned}
$$

In [12] and [13], Pop and Beaulieu gave excellent and detailed discussion on the PDF of the fading envelope, and the autocorrelation of the complex envelope of this model. They warned, however, that while their improved simulator is wide sense stationary, it may not model some higher-order statistical properties accurately. To further reveal the statistical properties of this model, we present the following correlation statistics of this model.

$$
\begin{aligned}
R_{X_c X_c}(\tau) &= \frac{1}{2N}\sum_{n=1}^{N}\cos\left(w_d\tau\cos\frac{2\pi n}{N}\right) && \text{(5a)}\\
R_{X_s X_s}(\tau) &= \frac{1}{2N}\sum_{n=1}^{N}\cos\left(w_d\tau\cos\frac{2\pi n}{N}\right) && \text{(5b)}\\
R_{X_c X_s}(\tau) &= \frac{1}{2N}\sum_{n=1}^{N}\sin\left(w_d\tau\cos\frac{2\pi n}{N}\right) && \text{(5c)}\\
R_{X_s X_c}(\tau) &= -\frac{1}{2N}\sum_{n=1}^{N}\sin\left(w_d\tau\cos\frac{2\pi n}{N}\right) && \text{(5d)}\\
R_{XX}(\tau) &= 2R_{X_c X_c}(\tau) + j2R_{X_c X_s}(\tau) && \text{(5e)}\\
R_{|X|^2|X|^2}(\tau) &= 1 + 4R_{X_c X_c}^2(\tau) + 4R_{X_c X_s}^2(\tau) + \frac{1}{N}. && \text{(5f)}
\end{aligned}
$$

We make three remarks based on (5): 1) The statistics of this modified model with $N = \infty$ are the same as the desired ones of the original Clarke's model. However, when $N$ is finite, the statistics of this model are different from the desired ones derived from Clarke's model; 2) the statistics of this model do not converge asymptotically to the desired ones when $N$ increases as discussed in [13] for the real part of $R_{XX}(\tau)$; 3) when $N$ is finite and odd, the imaginary part of $R_{XX}(\tau)$, along with $R_{X_cX_s}(\tau)$ and $R_{X_sX_c}(\tau)$, can significantly deviate from zero (the desired values), which implies that the quadrature components of this model are statistically correlated when $N$ is odd.

### *C. An Improved Rayleigh Fading Channel Simulator*

Based on the statistical analysis of Clarke's model and Pop and Beaulieu's modified model, we propose an improved Clarke's simulation model as follows.

*Definition 1:* The normalized lowpass fading process of a new statistical simulation model is defined by

$$Y(t) = Y_c(t) + jY_s(t) \tag{6a}$$

$$Y_c(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \cos(w_d t \cos\alpha_n + \phi_n) \tag{6b}$$

$$Y_s(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \sin(w_d t \cos\alpha_n + \phi_n) \tag{6c}$$

with

$$\alpha_n = \frac{2\pi n + \theta_n}{N}, \qquad n = 1, 2, \cdots, N, \tag{7}$$

where $\phi_n$ and $\theta_n$ are statistically independent and uniformly distributed over $[-\pi, \pi)$ for all $n$. It is noted that the difference between this improved model and Pop and Beaulieu's model is the introduction of random variables $\theta_n$ to the angle of arrival.

It can be shown that the first-order statistics of this improved model are the same as those of Pop and Beaulieu's model. However, some second-order statistics of this improved model are different, and they are presented below.

*Theorem 3:* The autocorrelation and cross-correlation functions of the quadrature components, and the autocorrelation functions of the complex envelope and the squared envelope of fading signal $Y(t)$ are given by

$$R_{Y_cY_c}(\tau) = \frac{1}{2} J_0(w_d\tau) \tag{8a}$$

$$R_{Y_sY_s}(\tau) = \frac{1}{2} J_0(w_d\tau) \tag{8b}$$

$$R_{Y_cY_s}(\tau) = 0 \tag{8c}$$

$$R_{Y_sY_c}(\tau) = 0 \tag{8d}$$

$$R_{YY}(\tau) = J_0(w_d\tau) \tag{8e}$$

$$R_{|Y|^2|Y|^2}(\tau) = 1 + J_0^2(w_d\tau) + \frac{1}{N}. \tag{8f}$$

*Proof:* The proof is similar to those of Theorems 1 and 2 given in [15], details are omitted for brevity. ■

As can be seen from Theorems 1 and 3, the correlation statistics of the improved model are the same as those of Clarke's model when both models have the same number of sinusoids. However, the *variances* of these correlations of the improved model are smaller than those of Clarke's model because the variance of the angle of arrival of the improved model is smaller than that of Clarke's model. Unfortunately, there are no closed-form expressions for the variances of these correlations of the improved model. Fig. 1 shows, as an example, some simulation results for the correlation variances of Clarke's model and the improved Clarke's model. Obviously, the variance of the autocorrelation of the complex envelope of our improved model is smaller than that of Clarke's model. This implies that the improved model converges faster than Clarke's model for a finite number of simulation trials. It is pointed out here that if we choose $\theta_n = \theta$ for all $n$, all the second-order statistics of $Y(t)$ will be the same as shown above, but the convergence of the ensemble average in simulation is slower.

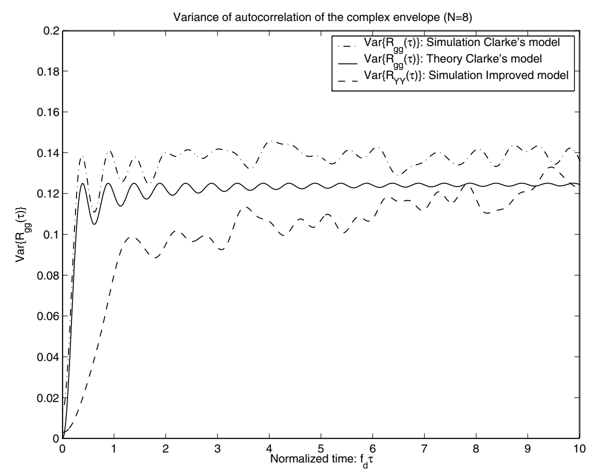


Fig. 1. Variances of autocorrelations of the complex envelope of Clarke's model and our improved model.

Before concluding this section, it is important to point out that the new simulation model can be directly used to generate uncorrelated fading samples for simulating frequency selective Rayleigh channels, MIMO channels, and diversity combing techniques. Let $Y_k(t)$ be the $k$th Rayleigh fading sample sequence given by

$$Y_k(t) = \sqrt{\frac{1}{N}} \sum_{n=1}^{N} \exp\left[ jw_d t \cos\left(\frac{2\pi n + \theta_{n,k}}{N}\right) + j\phi_{n,k} \right], \tag{9}$$

where $\theta_{n,k}$ and $\phi_{n,k}$ are mutually independent and uniformly distributed over $[-\pi, \pi)$ for all $n$ and $k$. Then, $Y_k(t)$ retains all the statistical properties of $Y(t)$ defined by eqns. (6). Furthermore, $Y_k(t)$ and $Y_l(t)$ are uncorrelated for all $k \neq l$, due to the mutual independence of $\theta_{n,k}$, $\phi_{n,k}$, $\theta_{n,l}$ and $\phi_{n,l}$ when $k \neq l$.

## III. A Novel Rician Fading Simulator

In this section, we present a statistical Rician fading simulation model and its statistical properties.

*Definition 2:* The normalized lowpass fading process of a new statistical simulation model for Rician fading is defined by

$$Z(t) = Z_c(t) + jZ_s(t) \quad (10a)$$
$$Z_c(t) = \left[Y_c(t) + \sqrt{K}\cos(w_d t\cos\theta_0 + \phi_0)\right]/\sqrt{1+K} \quad (10b)$$
$$Z_s(t) = \left[Y_s(t) + \sqrt{K}\sin(w_d t\cos\theta_0 + \phi_0)\right]/\sqrt{1+K}, \quad (10c)$$

where $K$ is the ratio of the specular power to scattered power, $\theta_0$ and $\phi_0$ are the angle of arrival and the initial phase, respectively, of the specular component, and $\phi_0$ is a random variable uniformly distributed over $[-\pi, \pi)$.

We present the correlation statistics of the fading signal, $Z(t)$, in the following theorem. The proofs are omitted.

*Theorem 4:* The autocorrelation and cross-correlation functions of the quadrature components, and the autocorrelation functions of the complex envelope and the squared envelope of fading signal $Z(t)$ are given by

$$R_{Z_cZ_c}(\tau) = \left[J_0(w_d\tau) + K\cos(w_d\tau\cos\theta_0)\right]/(2+2K) \quad (11a)$$
$$R_{Z_sZ_s}(\tau) = \left[J_0(w_d\tau) + K\cos(w_d\tau\cos\theta_0)\right]/(2+2K) \quad (11b)$$
$$R_{Z_cZ_s}(\tau) = K\sin(w_d\tau\cos\theta_0)/(2+2K) \quad (11c)$$
$$R_{Z_sZ_c}(\tau) = -K\sin(w_d\tau\cos\theta_0)/(2+2K) \quad (11d)$$
$$R_{ZZ}(\tau) = \left[J_0(w_d\tau) + K\exp(jw_d\tau\cos\theta_0)\right]/(1+K) \quad (11e)$$
$$R_{|Z|^2|Z|^2}(\tau) = \Big\{1 + J_0^2(w_d\tau) + 2K\left[1 + J_0(w_d\tau)\cos(w_d\tau\cos\theta_0)\right] + K^2 + \frac{1}{N}\Big\}/(1+K)^2. \quad (11f)$$

We now present the PDF's of the fading envelope $|Z|$ and phase $\Psi(t) = \arctan\left[Z_c(t), Z_s(t)\right]^1$.

*Theorem 5:* When $N$ approaches infinity, the envelope $|Z|$ is Rician distributed and the phase $\Psi(t)$ is uniformly distributed over $[-\pi, \pi)$, and their PDF's are given by

$$f_{|Z|}(z) = 2(1+K)z\cdot\exp\left[-K-(1+K)z^2\right] \times I_0\left[2z\sqrt{K(1+K)}\right], \quad z \geq 0 \quad (12a)$$
$$f_{\Psi}(\psi) = \frac{1}{2\pi}, \quad \psi \in [-\pi, \pi), \quad (12b)$$

where $I_0(\cdot)$ is the zero-order modified Bessel function of the first kind [18].

*Proof:* Since the sinusoids in the sums of $Y_c(t)$ and $Y_s(t)$ are statistically independent and identically distributed, $Y_c(t)$ and $Y_s(t)$ tend to Gaussian random processes as the number of sinusoids, $N$, increases without limit, according to the central limit theorem [20]. Moreover, since $R_{Y_cY_s}(\tau) = 0$ and $R_{Y_sY_c}(\tau) = 0$, $Y_c(t)$ and $Y_s(t)$ are independent. Therefore, $Z_c(t)$ and $Z_s(t)$ defined by (10) are also independent.

When the initial phase $\phi_0$ of the specular component is chosen, the conditional joint PDF of $Z_c(t)$ and $Z_s(t)$ is given by

$$f_{Z_c,Z_s}(z_c, z_s|\phi_0) = \frac{1+K}{\pi}\exp\Big\{-(1+K)\left[z_c - m_c(t)\right]^2 - (1+K)\left[z_s - m_s(t)\right]^2\Big\},$$

where $m_c(t) = \sqrt{\frac{K}{1+K}}\cos(w_d t\cos\theta_0 + \phi_0)$ and $m_s(t) = \sqrt{\frac{K}{1+K}}\sin(w_d t\cos\theta_0 + \phi_0)$.

Since the initial phase $\phi_0$ is uniformly distributed over $[-\pi, \pi)$, the joint PDF of $Z_c(t)$ and $Z_s(t)$ is given by

$$f_{Z_c,Z_s}(z_c, z_s) = \int_{-\pi}^{\pi} f_{Z_c,Z_s}(z_c, z_s|\phi_0)\cdot\frac{1}{2\pi}\cdot d\phi_0.$$

Transforming the Cartesian coordinates $(z_c, z_s)$ to polar coordinates $(z, \psi)$ with $z_c = z\cdot\cos\psi$ and $z_s = z\cdot\sin\psi$, we obtain the joint PDF of the envelope $|Z|$ and the phase $\Psi = \arctan(z_c, z_s)$,

$$f_{|Z|,\Psi}(z, \psi) = \frac{(1+K)z}{\pi}\cdot\exp\left[-K-(1+K)z^2\right] \times I_0\left[2z\sqrt{K(1+K)}\right], \quad z \geq 0,\ \psi \in [-\pi, \pi).$$

Then, the marginal PDF's of the envelope and the phase can be obtained by the following two equations

$$f_{|Z|}(z) = \int_{-\pi}^{\pi} f_{|Z|,\Psi}(z, \psi)d\psi, \quad f_{\Psi}(\psi) = \int_0^{\infty} f_{|Z|,\Psi}(z, \psi)dz.$$

This completes the proof. ■

We now highlight Theorem 5 with two remarks. First, both the fading envelope and the phase are stationary because their PDF's are independent of time $t$. This is very different from the previous Rician models [2], [9], where the PDF of the fading phase is a very complicated function of time $t$, and therefore the fading phase is not stationary as pointed out in [2]. Here, the fading phase of our new model is not only stationary but also uniformly distributed over $[-\pi, \pi)$. Secondly, the fading envelope and phase of our new Rician model are independent. As usual, the PDF's of the envelope and the phase of our Rician channel model include Rayleigh fading ($K = 0$) as a special case.

Two other important properties associated with the fading envelope are the *level crossing rate* (LCR) and the *average fade duration* (AFD). The LCR is defined as the rate at which the envelope crosses a specified level with positive slope. The AFD is the average time duration that the fading envelope remains below a specified level. We now present explicit formulas for the LCR and AFD for a general Rician fading channel whose specular component has *non-zero* Doppler frequency.

*Theorem 6:* When $N$ approaches infinity, the level crossing rate $L_{|Z|}$ and the average fade duration $T_{|Z|}$ of the new simulator output are given by

$$L_{|Z|} = \sqrt{\frac{2(K+1)}{\pi}}\rho f_d\cdot\exp\left[-K-(K+1)\rho^2\right] \times \int_0^{\pi} d\alpha\cdot\left[1 + \frac{2}{\rho}\sqrt{\frac{K}{K+1}}\cos^2\theta_0\cdot\cos\alpha\right] \times \exp\left[2\rho\sqrt{K(K+1)}\cos\alpha - 2K\cos^2\theta_0\cdot\sin^2\alpha\right], \quad (13a)$$
$$T_{|Z|} = \frac{1 - Q\left[\sqrt{2K}, \sqrt{2(K+1)\rho^2}\right]}{L_{|Z|}}, \quad (13b)$$

where $\rho$ is the normalized fading envelope level given by $|Z|/|Z|_{rms}$ with $|Z|_{rms}$ being the root-mean-square envelope level, and $Q(\cdot)$ is the first-order Marcum $Q$-function [21].

[1] The function $\arctan(x, y)$ maps the arguments $(x, y)$ into a phase in the correct quadrant in $[-\pi, \pi)$.

*Proof:* When $N$ approaches infinity, the fading envelope is Rician distributed as shown in Theorem 5. Using similar methods to those given in [17] for determining the LCR and in [19] for the AFD, one can prove eqns (13a) and (13b), respectively. ■

It is noted here that if $\theta_0 = \pi/2$, which means that the specular component has *zero* Doppler frequency, then the LCR given by (13a) has a closed-form solution. If $K = 0$, $Z(t) = Y(t)$ becomes a Rayleigh fading process; then both the LCR and the AFD have closed-form solutions.

## IV. Empirical Testing

Verification of the proposed fading simulator is carried out by comparing the corresponding simulation results for finite $N$ with those of the theoretical limit when $N$ approaches infinity. Throughout the following discussions, the newly proposed statistical simulators have been implemented by choosing $N = 8$ unless otherwise specified and all the ensemble averages for simulation results are based on 500 random samples unless otherwise specified.

### A. Evaluation of Correlation Statistics

We have conducted extensive simulations of the autocorrelations and cross-correlations of the quadrature components, and the autocorrelation of the complex envelope of both Rayleigh and Rician (with various Rice factors) fading signals. The simulation results of these correlation statistics match the theoretically calculated results with high accuracy. Therefore, we do not show these results here due to space limitations. The simulation results and the theoretically calculated results for the autocorrelation of the squared envelope of the fading signals are slightly different when $N = 8$ as can be seen in Fig. 2. The differences decrease if we increase the value of $N$.

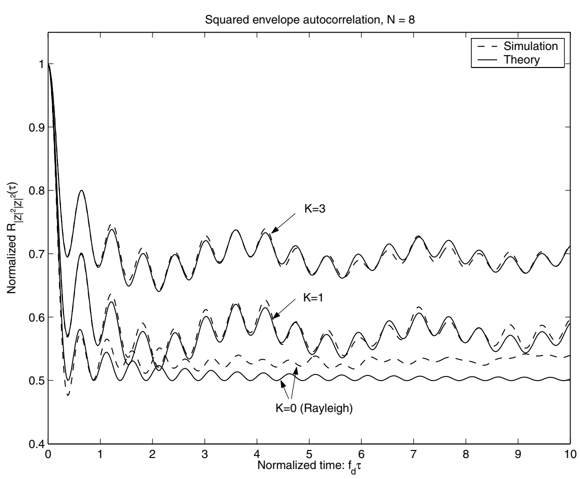


Fig. 2. The autocorrelation of the squared envelope $|Z(t)|^2$ and $\theta_0 = \pi/4$ for $K = 1$ and $K = 3$ Rician cases.

### B. Evaluation of Envelope and Phase PDF's

Figs. 3 and 4 show that the PDF's of the fading envelope and phase of the simulator with $N = 8$ are in very good agreement with the theoretical ones. It is also noted that when $N > 8$, these PDF's will have even better agreement with the theoretically desired ones.

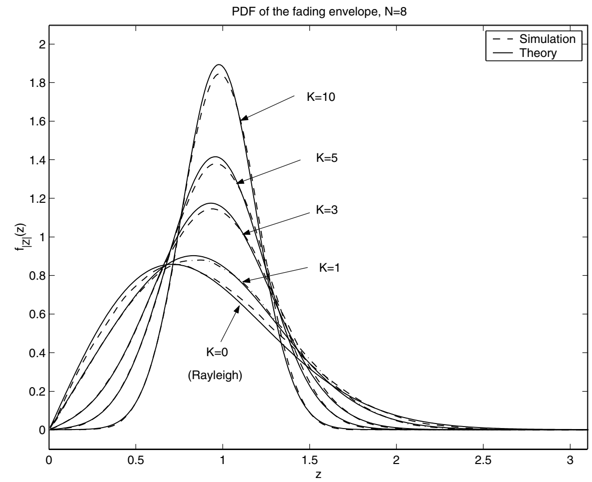


Fig. 3. The PDF of the fading envelope $|Z(t)|$.

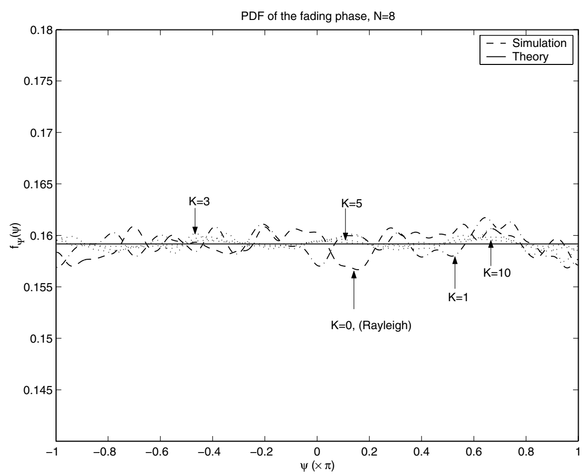


Fig. 4. The PDF of the fading phase $\Psi(t)$.

### C. Evaluation of the LCR and the AFD

The simulation results for the normalized level crossing rate (LCR), $\frac{L_{|Z|}}{f_d}$, and the normalized average fade duration (AFD), $f_d T_{|Z|}$, of the new simulators are shown in Figs. 5 and 6, respectively, where the theoretically calculated LCR and AFD for $N = \infty$ are also included in the figures for comparison, indicating good agreement in both cases. Again, if we increase the number of sinusoids, $N$, the simulation results for the case of finite $N$ approach the theoretical $N = \infty$ results.

For the region of $\rho < 0$ dB, it is interesting to note here that the average fade duration for $\theta_0 = 0$ (or $\theta_0 < \pi/4$) tends to be smaller for larger values of the Rice factor $K$. This is different from the AFD for $\theta_0 = \pi/2$, which tends to be larger with larger Rice factors.

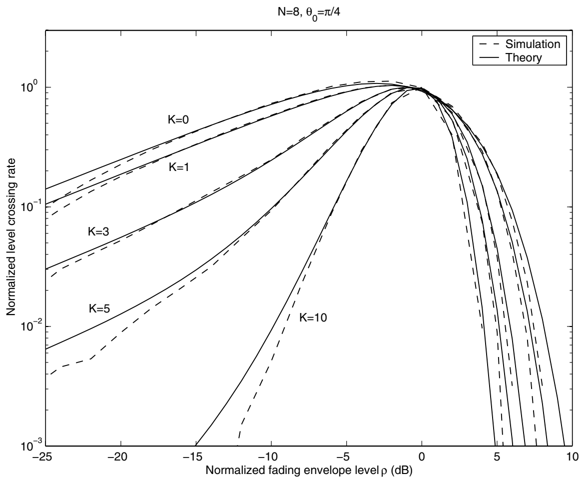


Fig. 5. The normalized LCR of the fading envelope $|Z(t)|$, where $\theta_0 = \pi/4$ for all $K > 0$ Rician fading.

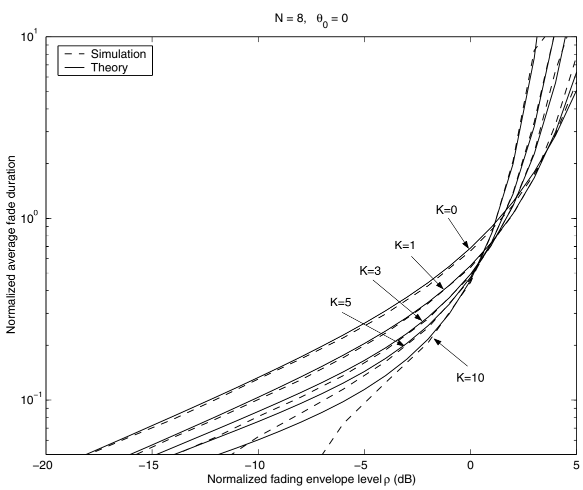


Fig. 6. The normalized AFD of the fading envelope $|Z(t)|$, where $\theta_0 = 0$ for all $K > 0$ Rician fading.

## V. Conclusion

In this paper, it was shown that Clarke's model with a finite number of sinusoids can be directly used for simulating Rayleigh fading channels, and its computational efficiency and second-order statistics are better than those of Jakes' original model [3] and as good as those of the recently improved Jakes' Rayleigh fading simulators [14], [16]. An improved Clarke's model was proposed to minimize the variance of the time averaged correlations of a fading realization from a single trial. A novel simulation model employing a random specular component was proposed for Rician fading channels. The specular (line-of-sight) component of this Rician fading model is a *zero-mean* stochastic sinusoid with a pre-chosen Doppler frequency and a random initial phase. Compared to all the existing Rician fading models, which have a *non-zero mean* deterministic specular component, the new model better reflects the fact that the specular component is random from time to time and from mobile to mobile. Additionally and importantly, the fading phase PDF of the new Rician fading model is independent of time and uniformly distributed over $[-\pi, \pi)$. All the theoretically predicted statistical properties of the new simulators have been verified by extensive simulation results. Excellent agreement was obtained in all cases.

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