$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}, C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

$$A \cdot B = C$$

$$\begin{aligned} C_{11} &= A_{11} \cdot B_{11} + A_{12} \cdot B_{21} \\ C_{12} &= A_{11} \cdot B_{12} + A_{12} \cdot B_{22} \\ C_{21} &= A_{21} \cdot B_{11} + A_{22} \cdot B_{21} \\ C_{22} &= A_{21} \cdot B_{12} + A_{22} \cdot B_{22} \end{aligned}$$

$$\begin{aligned} \text{I} &= (A_{11} + A_{22}) \cdot (B_{11} + B_{22}) \\ \text{II} &= (A_{21} + A_{22}) \cdot B_{11} \\ \text{III} &= A_{11} \cdot (B_{12} - B_{22}) \\ \text{IV} &= A_{22} \cdot (-B_{11} + B_{21}) \\ \text{V} &= (A_{11} + A_{12}) \cdot B_{22} \\ \text{VI} &= (-A_{11} + A_{21}) \cdot (B_{11} + B_{12})) \\ \text{VII} &= (A_{12} - A_{22}) \cdot (B_{21} + B_{22}) \end{aligned}$$

$$\begin{aligned} C_{11} &= \text{I} + \text{IV} - \text{V} + \text{VII} \\ C_{21} &= \text{II} + \text{IV} \\ C_{12} &= \text{III} + \text{V} \\ C_{22} &= \text{I} + \text{III} - \text{II} + \text{VI} \end{aligned}$$

$$C_{11} = (A_{11} + A_{22}) \cdot (B_{11} + B_{22}) + A_{22} \cdot (-B_{11} + B_{21}) - (A_{11} + A_{12}) \cdot B_{22} + (A_{12} - A_{22}) \cdot (B_{21} + B_{22}) C_{21}$$

$$C_{11} = A_{11}B_{11} + A_{11}B_{22} + A_{22}B_{11} + A_{22}B_{22} - A_{22}B_{11} + A_{22}B_{21} - A_{11}B_{22} - A_{12}B_{22} + A_{12}B_{21} + A_{12}B_{22} - A_{22}B_{21} - A_{22}B_{22}$$

$$C_{11} = A_{11}B_{11} + A_{12}B_{21}$$

# 1 Winograd

$x_1y_1 + x_2y_2 = (x_1 + y_2)(y_1 + x_2) - x_1x_2 - y_1y_2$
$x = (x_1, \cdots, x_n), y = (y_1, \cdots, y_n)$

$$\xi = \sum_{j=1}^{\lfloor n/2 \rfloor} x_{2j-1} \cdot x_{2j}$$

$$\eta = \sum_{j=1}^{\lfloor n/2 \rfloor} y_{2j-1} \cdot y_{2j}$$

$$\langle x, y \rangle = \begin{cases} \sum_{j=1}^{\lfloor n/2 \rfloor} (x_{2j-1} + y_{2j})(x_{2j} + y_{2j-1}) - \xi - \eta & \text{if } n \text{ is even} \\ \sum_{j=1}^{\lfloor n/2 \rfloor} (x_{2j-1} + y_{2j})(x_{2j} + y_{2j-1}) - \xi - \eta + x_n y_n & \text{if } n \text{ is odd} \end{cases}$$