

## 1 INTRO

## 2 GEOMETRIE

We'll start with geometric symmetries as they are the simplest to grasp.

[ Intro ]

To mathematically formulate the concept, we will think of symmetries as actions to perform on an object, like this square. The simplest action, is to take this square, do nothing and put it back down. Another action could be to flip it along an axis, or to rotate it around its center by 90 degrees.

[ Cyclic Groups ]

Let's focus our attention on the simplest class of symmetries: those generated by a single rotation. We will gather the symmetries in a group  $G$ , and denote that it is generated by a rotation  $r$  with these angle brackets.

Take this pentagon as an example. By applying the rotation action 5 times, it is the same as if we had not done anything, furthermore, if we act a sixth time with  $r$ , it will be the same as if we had just acted with  $r$  once. Thus the group only contain the identity and the powers of  $r$  up to 4.

In general, groups with this structure are known as the "Cyclic Groups" of order  $n$ , where the action  $r$  can be applied  $n - 1$  times before wrapping around.

[ Dihedral Groups ]

Okay that was not difficult, now let's spice this up a bit. Consider this group for a square, generated by two actions: a rotation  $r$  and a reflection  $\sigma$ . Because we have two actions we have to write in the generator how they relate to each other.

Let's analyze this expression. Two reflections are the same as the identity. Four rotations are the same as the identity, and a rotation followed by a reflection, twice, is the same as the identity.

This forms a group with 8 possible unique actions. This too can be generalized to an  $n$ -gon, and is known as the "Dihedral Group" of order  $n$ .

[ Symmetrische Gruppe ]

[ Alternierende Gruppe ]

3 ALGEBRA

4 KRYSTALLE