

# FM AND BESSEL ZEROS

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## FM AND BESSEL ZEROS

**ACHIEVEMENTS:** calibration of the frequency deviation of an FM transmitter using the method of Bessel Zeros.

**PREREQUISITES:** completion of the experiments entitled *Armstrong's phase modulator*, and *FM deviation multiplication* in this Volume; a knowledge of the relationships between the phase deviation and the spectrum of a PM signal. See **Appendix C** to this text for Tables of Bessel Coefficients.

**EXTRA MODULES:** 100 kHz CHANNEL FILTERS (version 2); FM UTILITIES, SPECTRUM UTILITIES.

## PREPARATION

### ***introduction***

This experiment investigates methods of deviation calibration of a PM transmitter by observation of the spectrum. It includes the method of 'Bessel zeros'.

The outcome of the experiment could be a calibration curve, showing the position of the modulator deviation control versus  $\beta$ , the deviation of the modulator<sup>1</sup>.

This curve could already have been obtained in the experiment entitled *Armstrong's phase modulator*, by measuring the ratio of the DSBSC to carrier amplitudes out of the ADDER. From this ratio the magnitude of the spectral components could have been deduced by *calculation*. But in *this* experiment you will be examining the spectrum itself, and from this working backwards to determine the phase deviation.

*Make sure you appreciate the difference between the two methods.*

The model required to generate a PM signal is that used in the experiment entitled *FM deviation multiplication*. Refer to that experiment for setting up details.

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<sup>1</sup> the *principle* of the method is what will be learned. You cannot actually plot the curve, since TIMS knobs are not graduated

Figure 1 shows the arrangement in simplified block diagram form, and Figure 2 shows patching details.

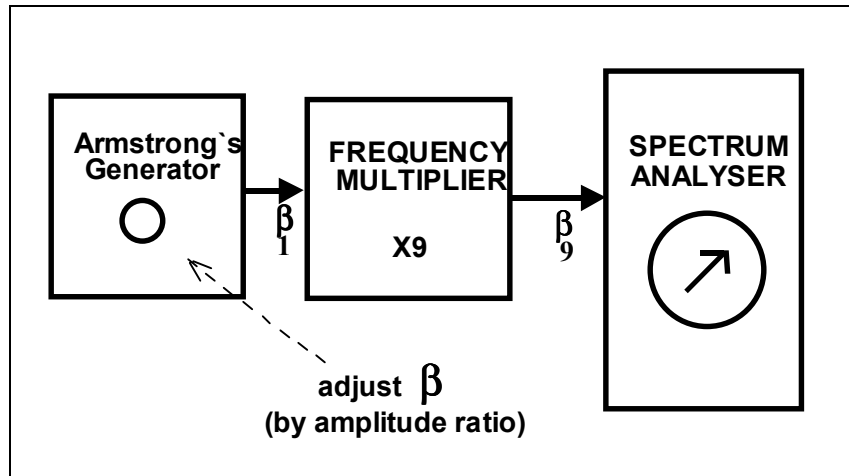


Figure 1: the measurement set-up

# EXPERIMENT

The block diagram of Figure 1 is shown modelled in Figure 2 below. This model was examined in the experiment entitled *FM deviation multiplication*, so the setting up details here will be brief.

*T1* set up the model illustrated in Figure 2. Choose a suitable message frequency (this will be below 1 kHz?). Adjust the phasing in the Armstrong modulator, using the envelope as a guide (see the experiment entitled *Armstrong's phase modulator*).

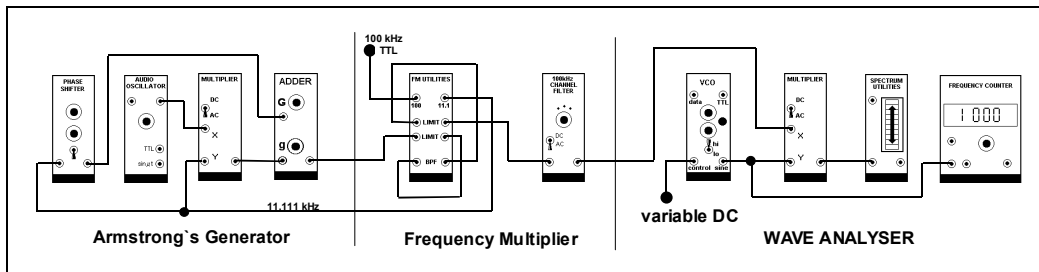


Figure 2: patching details

To ensure the modulator does not introduce spectral distortion, ensure at all times that the phase deviation at the ARMSTRONG MODULATOR is kept well below

1.0 radian. This means that the ratio of DSBSC to carrier, at the ADDER output, must remain less than unity.

Note that the phase deviation at the FREQUENCY MULTIPLIER output will be the Armstrong phase deviation multiplied by a factor of 9.

Let us denote the phase deviation at the ARMSTRONG MODULATOR as  $\beta_1$  and the phase deviation at the FREQUENCY MULTIPLIER output as  $\beta_9$ , as in Figure 1.

## **spectral components**

You will be using the WAVE ANALYSER to measure the amplitude and frequency of the spectral components of various signals. For this experiment the *absolute* amplitudes of the spectral components are of secondary importance. What will interest you is their *relative amplitudes*. Thus it is not necessary to calibrate the amplitude sensitivity of the WAVE ANALYSER.

## **locate the 'carrier'**

*T2 with  $\beta_1$  set to zero, locate the unmodulated carrier with the WAVE ANALYSER. It should be at about the TIMS ANALOGUE REFERENCE LEVEL.*

Spectral amplitudes are typically quoted with respect to the amplitude of the unmodulated carrier. Thus it is convenient to set this component to full scale on the measuring equipment. This can be done by tuning to the carrier and manually setting the on-board adjuster RV1, labelled 'SCALING', for this condition.

*T3 adjust the reference signal to full scale deflection on the meter of the SPECTRUM UTILITY module.*

*T4 check that there are no other components of significance within 10 kHz of the carrier.*

*T5 set  $\beta_1$  to about 0.16 and search for components of significance within 10 kHz of the carrier. Record the frequency and amplitude of all components found.*

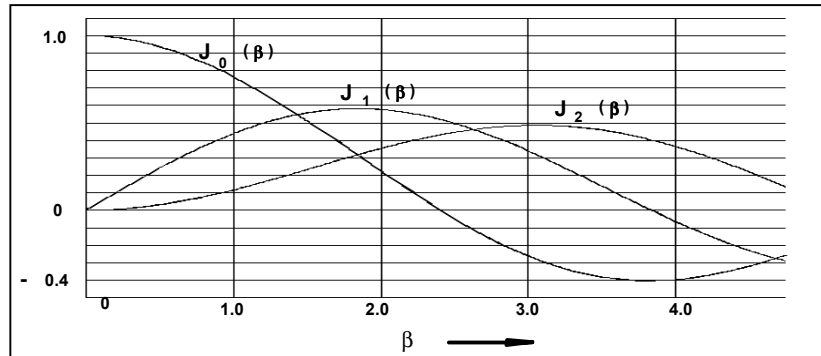
*The component at the carrier frequency  $\omega$ , and the components at  $(\omega \pm \mu)$ , should have been of about equal magnitude.*

This fact can be checked by reference to the curves of Figure 3 below. For:

$$\beta_1 = 0.16$$

then

$$\beta_0 = 1.44$$



**Figure 3: Bessel function plots**

You will note that  $J_0(\beta)$  is approximately equal to  $J_1(\beta)$  when  $\beta_0$  is about 1.4. In fact they approach equality for  $\beta = 1.435$ .

But the amplitude of the carrier is proportional to  $J_0(\beta)$ , and that of the first pair of sidebands is proportional to  $J_1(\beta)$ .

So here is a way of calibrating the ARMSTRONG MODULATOR phase deviation control. Adjust the DSBSC amplitude, at the output of the ADDER in the ARMSTRONG GENERATOR, until these two components are equal.

You have now set

$$\beta_0 = 1.435$$

Note that the method did not involve the measurement of an absolute amplitude, but rather the matching of two amplitudes to equality. So the amplitude sensitivity of the WAVE ANALYSER need not be calibrated.

This amplitude matching method can be applied to determine other values of  $\beta_0$ . From the curves of Figure 3 one could suggest the following pairs:

<i>components</i>	$\beta_0$	$\beta_1$
carrier and second	1.85	0.21
first and second	2.6	0.29
carrier and second	3.8	0.42

**Table 1: equal amplitude sidefrequency pairs**

**T6** check some or all of the pairs of sidefrequencies listed in Table 1. These will give other points on the curve of  $\beta$  versus the modulator phase deviation control.

## ***the method of Bessel zeros***

So far the calibration points have been obtained by equating the amplitudes of two spectral components.

There is an even more precise method of obtaining points on the calibration curve. Not only is an absolute amplitude reading not required, but there is only a single measurement to make - and this is a *null* measurement. There is no need for a calibrated instrument.

This is *the method of Bessel zeros*.

Note from Figure 3 that the Bessel functions are oscillatory (but not, incidentally, periodic). In fact they are *damped* oscillatory, which means that successive maxima are monotonically decreasing. But for the moment the important property is that they are oscillatory about zero amplitude, which means that there are values of their argument for which they become zero.

*There are precise, and multiple values, of  $\beta$ , for which the amplitude of a particular spectral component of an angle modulated signal falls to zero.*

If you can find when the amplitude of a particular spectral component falls to zero, you have a precise measure of  $\beta_0$ , and another point on the calibration curve. It is easier to find a *single* zero, by trimming of the ARMSTRONG MODULATOR phase deviation control <sup>2</sup>, than it is to adjust the amplitudes of *two* components to equality.

## ***looking for a Bessel zero***

One would normally think of using a WAVE ANALYSER when looking for Bessel zeros. So in the first instance this will be done.

### ***using the WAVE ANALYSER***

Table 2 below shows some particular Bessel zeros which you can use experimentally. These can be checked by reference to the curves of Figure 3.

**Table 2**

<b>Bessel coefficient</b>	<b>side frequency</b>	<b>first zero</b>	<b>second zero</b>
$J_0(\beta)$	central carrier	2.41	5.52
$J_1(\beta)$	first pair	3.83	7.02
$J_2(\beta)$	second pair	5.13	8.4

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<sup>2</sup> the ADDER gain control  $g$  which adjusts the DSBSC amplitude

Each Bessel zero will give a point on the calibration curve.

For a multiplication factor of 9, as you are using, and the Armstrong modulator as the source of the phase deviations,  $\beta_9$  is restricted to the range 0 to about 3 radians<sup>3</sup>. So only the first carrier zero, and the first sidefrequency pair zero, are available to you. But these are quite sufficient to demonstrate the method.

Note that it is necessary to keep track of *which* zero one is seeking. This is relatively simple when finding the first or second, but care is needed with the higher zeros. The problem will not arise in this experiment.

*T7 whilst monitoring the amplitude of the component at carrier frequency, increase the phase deviation control on the ARMSTRONG MODULATOR from zero until the amplitude is reduced to zero. Measure the amplitude ratio of the DSBSC and carrier at the ADDER output. This should be about  $(2.4/9.0) = 0.27$ . Check this expected value against your tables. Explain any disagreement between measured and expected values.*

*T8 with  $\beta_9$  as for the previous Task, locate either of the first pair of sidefrequencies ( $100 \text{ kHz} \pm \text{message frequency}$ ). Increase the phase deviation control on the ARMSTRONG MODULATOR until the amplitude of the chosen component is reduced to zero. Measure the amplitude ratio of the DSBSC and carrier at the ADDER output. This should be about  $(3.8/9.0) = 0.42$ . Explain any disagreement between measured and expected values.*

## **without a WAVE ANALYSER**

In practical engineering one is often (always ?) looking for ways and means of simplifying procedures, and avoiding the use of expensive equipment, especially when in the field.

The Bessel zero method of frequency deviation calibration is extremely precise, but appears to need an expensive SPECTRUM ANALYSER for its execution. But this need not be so, especially if one is content to use only the zeros of the carrier component.

All that is needed is an oscillator close to the carrier frequency, a multiplier, and a pair of headphones.

The principle is that the unmodulated FM signal and the local oscillator are multiplied together, and the difference component of the product monitored with the headphones. The local oscillator is adjusted to give a convenient difference frequency - say about 1 kHz.

Now, while concentrating on this tone, the transmitter frequency deviation is increased from zero. The message should be a single tone. The amplitude of the 1 kHz tone will decrease, until it falls to zero when  $\beta$  is at the first zero of  $J_0(\beta)$  - approximately  $\beta = 2.4$ .

It is true that under this condition there will be other tones present in the headphones. But by 'suitable' choice of message frequency these will lie above the 1 kHz tone

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<sup>3</sup> distortion is discussed in the experiment entitled *Armstrong's frequency modulator*.

that is being monitored, and one can, with experience, ignore them. The key to the method lies in choosing a 'suitable' message frequency.

Suffice to say, the method is used with success in practice.

There is no need for a true MULTIPLIER; almost any non-linear device will do, typically an overloaded transistor amplifier which will generate intermodulation products, including the wanted 1 kHz difference component.

*T9 demonstrate the method of setting a Bessel zero, using only a listening device and a non-linear element (there is a rectifier in the UTILITIES module), as described above.*

## **TUTORIAL QUESTIONS**

There are suitable tutorial questions in the Chapter entitled *Analysis of the FM spectrum*.